Sabancı Üniversitesi

ECON 488/688

Matchings and Markets

Spring 2022

11:40-14:30 Wednesday

Online

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COURSE DESCRIPTION

Matchings and Markets is about markets where individuals are matched with individuals or firms or items, typically across two sides, as in marriage, employment, university entrance, housing. Such diverse markets are looked at through a common game theoretic model. The aim is to identify outcomes that have nice properties such as stability, optimality or fairness and to design mechanisms to realize such outcomes.

Keywords : coalitional games; competitive equilibrium; stable (core) outcomes; existence, optimality, partial order lattice structures; algorithms, constructive procedures, auctions; strategic properties, mechanisms; institution and market design.

The course is open to all students, in particular from Economics, Computer Science, and Industrial Engineering, as Econ 488 for undergraduates and as Econ 688 for graduate students.

Enrollment is subject to special approval for undergraduates. Students are asked to contact the instructor by email or whatsapp.

The subject has evolved from the original paper by D. Gale and L. Shapley (1962) titled “College Admissions and the Stability of Marriage” and the paper by M. Beckmann and T. Koopmans (1958) titled “Assignment Problems and the Location of Economic Activities”.

The Economics Nobel Prize in 2012 was awarded to *Matchings and Market Design* – to two of its original contributors L. Shapley and A. Roth – for remarkable success in the analysis and design of actual matching markets. Some current examples are (1) student placement in schools, (2) labor markets between workers and firms, e.g., doctors and hospitals, (3) organ donation, in which patients are matched to potential donors, and (4) rent-controlled housing markets, (5) multi-object auctions. The Economics Nobel Prize in 2020 was awarded to P. Milgrom and R. Wilson for “improvements in auction theory and inventions in new auction formats.”

The subject has more recently attracted the interest of Computer Science : see the conference series MATCH-UP and *Algorithmic Game Theory*.

This course has no prerequisite. On the other hand, it requires mathematical thinking and rigor. Theorems proofs algorithms play an essential role.

SuCourse will be used substantially with weekly announcements and updates. Articles and notes to be studied or referred to, as well as the classical textbook "Two-Sided Matching: A Study in Game-Theoretic Modeling and Analysis" by A. Roth and M. Sotomayor (1990), will be uploaded in the course site.

OUTLINE

The course will start with “one-to-one” matching, treating in parallel the Discrete and the Continuous Model, and studying Existence, Optimality and Lattice Structure of Stable Allocations; Algorithms and Procedures; Strategyproofness.

The course will then take up “one-to-many” and “many-to-many” matching, with emphasis on understanding preference domains where “one-to-one” matching properties continue to hold.

Later and alongside in the course, various selections from the extended literature will be studied: School Choice, Matching with Contracts, Fair Allocations, Multi-object Auctions, Kidney Exchange, ,…

COURSEWORK AND GRADING

There will be one exam. In addition, each student will make a presentation or submit a paper on a topic of her choice approved by the professor. Attendance is required. Active participation is called for.

The letter grade will be based on exam (50 %), presentation or term paper (30%), participation (20%).

APPENDIX

I. One-To-One Markets

1. (i) The Marriage Model (Gale Shapley 1962) : Existence of stable matchings; Lattice structure; Optimality. Standard proofs.

   (ii) The Sellers and Buyers Model with Transferable Utility (Shapley Shubik 1970) and Nontransferable Utility (Alkan Gale 1990) : Existence of stable matchings, equivalence to core allocations and to competitive price equilibria; Lattice structure; Optimality. Proofs elementary and constructive. Multiobject auctions (Demange Gale Sotomayor 1986, Alkan 1992). Link with envyfree allocations of indivisible items (Alkan Demange Gale 1991).

2. Strategyproofness of Optimal Mechanisms (Dubins Freeman 1981, Gale Sotomayor 1985, Leonard 1982)

II. One-To-Many and Many-To-Many Markets

1. To what extent and when the existence-structure-optimality results hold for one-to-many and many-to-many models in Gale Shapley 1962, Kelso Crawford 1982, Roth 1982, 1984, 1985, Gul Stachetti 1999, 2000, Alkan 2002, Fleiner 2003, Alkan Gale 2003, Hatfield Milgrom 2005. A unified look at alternative constructive procedures. Tarski Fixed Point Theorem perspective. The *substitutability* and *size monotonicity* conditions. Comparative statics.

References

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A. Kelso, V. Crawford, 'Job matching, coalition formation and gross substitutes', *Econometrica* 50, 1982

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L. Shapley, M. Shubik, "The Assignment Game I: The Core", *International Journal of Game Theory*, 1, 2, 1971